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COMPARISON OF DYNAMIC PERFORMANCES OF TWO-MASS DRIVE SYSTEM WITH PI AND FL CONTROLLERS IN TRAJECTORY TRACKING PROBLEM

The paper deals with the analysis of dynamic properties of DC drive system with elastic joint and different speed controllers used in the trajectory tracking problem. The cascade control structure with two speed feedbacks was analysed. The dynamical properties of the drive system with classical PI and Fuzzy-Logic speed controllers were compared for different parameters and operation modes of the drive as well as for different reference speed trajectories. Parameters of the classical PI and FL speed controllers were optimised using the same control indexes and Genetic-Gradient Algorithm. Simulation results of the analysed drive system were demonstrated and evaluated.

1. INTRODUCTION

In the recent years fuzzy logic (FL) control appeared a great field of interest in many technical applications, also in electrical drives [1]-[5]. It has happened due to its advantages in control of nonlinear plants where dynamics changes with the operating point of the system. Besides, the other plant uncertainties or disturbances can appear in the control process, what require nonlinear controllers, which are complicated and difficult from the design methods’ point of view. Fuzzy-logic control simplifies the solution of such problems in many cases. FL controllers are specially designated to control problems of nonlinear, non-stationary or ill-defined systems. But even for linear plant the control problem becomes nonlinear one, if the control index is nonlinear [5]. In such case the linear controller is not effective and FL controller with nonlinear control surface can give much better results.

In this paper the effectiveness of fuzzy-logic controller and classical PI controller
was compared for the two-mass DC drive system with an elastic joint. The elastic strains usually cause the vibration in the mechanical system, which influence the electrical and electromechanical variables of the motor. Such systems were described in many papers where analytical design methods for the adjustment of PID controllers were proposed for the compensation of the vibration and minimisation of motor transients. Recently some attempts are observed in the field of FL application for the optimal control problem of the drive systems with elastic joints [10]. However, analytical tuning methods for FL controllers are still missing [2]-[4] and the main problem is the adjustment method of FL controller in such system. Usually the heuristic method based on trial and error is used. Besides, in all papers the different adjustment methods are used for classical PI controllers and FL controllers, thus the drive system performance comparison is rather difficult.

In this paper the PI or FL speed controllers for the drive system with the elastic joint were designed for the same control index and using the same adjustment method, based on Genetic Algorithms. So the comparison of the dynamical performance of the two-mass system with different controllers was possible, for the different operation modes of the drive system.

2. THE DRIVE SYSTEM DESCRIPTION

The considered two-mass drive system was demonstrated in Fig. 1. The permanent magnet DC motor was taken into account as a driven motor. But in fact, the results obtained for the speed control loop can be also used for any drive motor with suitable torque control method, like vector-controlled AC motors.

\[ m_1(t) \quad T_{M1}(t) \quad m_2(t) \quad T_{M2}(t) \quad m_0(t) \]
\[ \omega_m(t) \quad \omega_s(t) \]

Fig. 1. Schematic diagram of the drive system with elastic joint
Rys. 1. Schemat układu dwumasowego

The DC motor drive system (for constant flux \( \Psi_f \)) with elastic joint is described by the following equations in per unit system:

\[ T_e \frac{di_s}{dt} = -i_s + K_i(u_a - \Psi_f \omega_m) \]  \hspace{1cm} (1)
\[
\frac{d\omega_m(t)}{dt} = \frac{1}{T_{M1}} \left( m_a(t) - m_s(t) - m_f(t) \right),
\]
\[
\frac{d\omega_r(t)}{dt} = \frac{1}{T_{M2}} \left( m_s(t) - m_L(t) - m_f(t) \right),
\]
\[
\frac{d\varphi(t)}{dt} = \frac{1}{T_C} \left( \omega_m(t) - \omega_r(t) \right),
\]
\[
m_s = \begin{cases} 
\phi(t) - \frac{\varepsilon}{2} \text{sgn}(\varphi) + h \frac{d}{dt} \left( \varphi(t) - \frac{\varepsilon}{2} \text{sgn}(\varphi) \right) & \text{for } \varphi > \frac{\varepsilon}{2} \\
0 & \text{for } \varphi < \frac{\varepsilon}{2}
\end{cases}
\]

where: \( u_a, i_a \) – armature voltage and current, \( K_t \) – gain factor of the motor, \( \Psi_f \) – excitation flux, \( m_e, m_L \) – electromagnetic and external load torque, \( m_f \) – friction torque, \( m_S \) – torsion torque in the elastic joint, \( \omega_m \) – angular rotor speed, \( \omega_L \) – angular speed of loading machine, \( T_e \) – electromagnetic time constant of the armature winding, \( T_{M1} \) – mechanical time constant of the motor, \( T_{M2} \) – mechanical time constant of the load machine, \( T_C \) – time constant of the elastic joint, \( h \) – damping factor of the elastic joint, \( \varepsilon \) – the backlash.

The control structure of this two-mass system, analysed in the paper is demonstrated in Fig.2.

Fig. 2. Schematic diagram of the control structure for the 2-mass drive system with suitable feedbacks
Rys. 2. Schemat struktury sterowania układu dwumasowego z dodatkowym sprzężeniem zwrotnym
In the analysed structure the friction torque described according to [12] was taken into account:

\[ m_f = \frac{\omega}{d} \frac{\omega}{\omega^2 + \frac{1}{d}} + k \left[ \frac{2}{\pi} \arctan(\gamma \omega) \right]^3 \]  

(6)

The speed control system of DC drive has the most used cascade closed loop structure with PI armature current controller and PI speed controller which are usually adjusted according to well known modulus and symmetry criteria [11]. In the considered case the supervisor speed controller was a classical PI or FL controller. This main control loop contains two speed feedbacks: the first one - from the motor speed and the second one - from the load machine (marked with a dashed line). In [7] different, additional feedbacks were proposed.

3. CLASSICAL LINEAR PI AND FL CONTROLLER

In the classical PI controller the output is simply the sum of its proportional and integral part multiplied by suitable gain factors \( K_P \) and \( K_I \). The control surface of this controller is thus a two-dimensional plane with two degrees of freedom, depending on these \( K_P \) and \( K_I \) coefficients. So the change of these coefficients enables only the change of the plane slope. A fuzzy logic controller contains much more parameters which can change its control surface. These are: rule base, membership functions and scaling factors \( k_e, k_{de}, k_{du} \). The number of freedom degrees depends on the controller parameter number an is much bigger than for the linear PI controller, what was demonstrated in Fig. 3.

The flexibility in the control surface shaping is the most important feature of FL controller, because it enables the best adjustment of the controller properties to the controlled plant. But it advantage is connected with the increase of changed parameters of the controller, what significantly complicates the adjustment procedures and execution time of the numerical control algorithm in the practical implementation.

Moreover, analytical adjustment methods of the FL do not exist. So it is necessary to design the structure of such controller as simple as possible, to obtain the required control accuracy and simultaneously the simplicity of practical realisation of the control structure.
4. ADJUSTMENT OF PI AND FL CONTROLLERS

The classical PI controller has only two adjusted parameters $K_P$ and $K_I$. So in the case of the well identified control plan it is easy to adjust them using well known methods, as: different integral control indexes or symmetry and modulus criteria [11]. But it should be mentioned, that always some simplified assumption are made during such adjustment procedures, e.g. in the case of DC motor drive it is assumed that back electromotive force of the motor is slowly variable disturbance because of $T_M>>T_e$ and the friction influence is neglected, what enables an easy calculation of the controller parameters [11]. So its clear that in the real system the dynamical properties of the drive are not optimal.

In the case of FL controllers the adjustment problem is much more complicated due to greater number of controller parameters (rule base, membership functions and scaling factors $k_o$, $k_{de}$, $k_{du}$) and lack of analytical design methods. The adjustment method based on an expert knowledge or on equivalences between classical linear and FL...
controllers [3] not always gives the optimal results. The one of solution of this problem could be the application of Genetic Algorithms. They can be used for the optimisation of scaling factors (what is the simplest and frequently used solution), shape and number of membership functions as well as rule base (a solution used in the case of FL controllers with big rule base). In this paper the following methodology of the controllers’ adjustment was applied:

- for the classical PI controller two controller parameters $K_P$ and $K_I$ as well as a gain coefficient in the additional speed feedback taken from the speed $\omega_L$ of the loading machine were adjusted using the suitable optimisation procedure minimising the chosen control index;

- for the FL controller, with the same control index applied, the very simple rule base was chosen in the first step of the optimisation procedure. This rule base was symmetrical, to have the same operation condition of the drive system in both speed directions. Then the scaling factors were adjusted. In the last step the shape of membership functions together with scaling factors were optimised finally, to obtain the minimal value of the control index. Such optimisation sequence has ensured the minimal execution time of the optimisation procedure.

In the optimisation procedure the combined technique was applied: GA was used for determination of the starting point of the gradient algorithm only and then the suitable controllers’ parameters were searched using the gradient method. Such hybrid genetic-gradient algorithm (GGA) is characterised by high efficiency in the search of global extreme surrounding (where genetic algorithm is the most effective) as well as in searching of optimal solution (where the gradient algorithm is very fast).

In the most cases described in technical papers, for 2-mass system control the classical PI controllers were used [6]-[8]. The main goal of this paper was the comparison of the dynamic behaviour of the 2-mass DC drive system with the linear PI speed controller and FL speed controller, both designed using the same control indexes as well as the same genetic-gradient optimisation procedure. The following control indexes were taken into account:

- $K_1$ control index, using well known integral criterion:

$$K_1 = \min \int_0^\infty \left( \omega_{\text{ref}} - \omega_L \right)^2 dt$$

- $K_2$ control index, using the same integral criterion and additional limit for the loading machine speed $\omega_L$:

$$K_2 = \begin{cases} \min \int_0^\infty \left( \omega_{\text{ref}} - \omega_L \right)^2 dt \\ \omega_L \leq \omega_{\text{ref}} \end{cases}$$
For such defined control task, the genetic-gradient optimisation algorithm was applied to obtain the proper adjustment of both types of the speed controllers.

5. INFLUENCE OF THE CONTROLLER TYPE TO DYNAMICAL PROPERTIES OF THE DRIVE SYSTEM

The simulation tests were performed for the 2-mass DC drive system with two speed control loops, as presented in Fig. 2.

![Graphs showing drive system behaviour](image)

Rys. 4. Przebiegi zmiennych układu dwumasowego w przypadku wskaźnika jakości sterowania $K_I$ i liniowego regulatora PI dla zadanej prędkości a,b) $\omega_{ref} = 1$, c,d) $\omega_{ref} = 0.1$

The various operation modes were simulated, but the most interesting was the behaviour of the system in the low speed region for the step speed reference and for the reference trajectory tracking task. In the following figures transient responses of the motor speed as well as the loading machine speed to the changes of the speed reference signal are presented. The other variable as electromagnetic and torsion torque
transients of the drive system are also demonstrated.

The drive system behaviour in the case of $K_I$ control index (7), for PI and FL speed controllers used in the control structure are demonstrated in Fig. 4 and Fig. 5 respectively. Both controllers were adjusted for nominal motor speed and then dynamical properties of the drive system were checked in the whole speed range.

For lower speed reference values the behaviour of the drive system is getting worse, especially for very low speed (Fig. 4c and Fig. 5c). The overshoots of angular speed of loading machine as well as motor speed occur. In Fig. 6a,b the comparison of angular speed of loading machine $\omega_k$ for nominal and very low reference speed value are demonstrated for both controllers.

![Fig 5. The drive system behaviour in the case of $K_I$ control index and FL controller: a,b) $\omega_{ref} = 1$, c,d) $\omega_{ref} = 0.1$](image)

It is seen from all transients, that the two-mass drive system with PI and FL controllers, adjusted according to the $K_I$ control index using the same optimisation algorithm (GGA), have similar properties for nominal speed reference. The advantage of FL controller is observed when the speed reference is changed to very low range.

![a) b) c) d)](image)
Next the similar adjustment procedure for classical PI and FL speed controllers was performed using $K_2$ control index (8). For nominal speed reference the behaviour of the drive system with FL controller is similar to the system with PI classical controller, because the adjustment procedure (control index and the optimisation algorithm) was the same. It should be mentioned that in the case of FL controller the minimal value of the control index was slightly smaller, what resulted in slightly faster drive response. But for lower speed reference the properties of the drive system with FL speed controller are much better: the overshoots in the motor and load speed as well as transients of the torsion torque and armature current reach smaller values on transients and the drive speed settling time were much shorter. The angular speed limit of the loading machine $\omega_{\text{ref}} \leq \omega_{\text{ref}}$ was achieved for the wide speed range in the case of FL controller only. For classical PI controller the overshoot in $\omega_L$ appeared even for 50% of nominal
speed reference, when in the case of FL controller this overshot, less than \(0.2\omega_{\text{ref}}\), was observed only in a very low speed region. It is seen from simulation tests, that in the case of equal inertia torque of the motor and loading machine this overshoot was equal zero even for 15% of nominal speed reference. Transients of the torsion torque were also smaller what is very important for mechanical couplings. For better illustration of these features the comparison of angular speed of loading machine \(\omega_L\) for nominal and very low reference speed value are demonstrated in Fig. 6c,d, for both speed controllers adjusted according control index \(K_2\).

FL controller ensures much better dynamical performances of the drive system, especially in the low speed region. It means, that FL controller, adjusted for nominal operation condition \((\omega_N)\), is much robust to changes of the drive operating point. It is clear that for more nonlinear control index, the nonlinear (FL) controller gives better results.

6. CASE OF SPEED REFERENCE TRACKING

Both speed controllers were checked also in the case of speed reference tracking task. Their parameters were the same as in the previous tests – optimised for the step speed reference and the same control indexes.

In Fig.7 and Fig.8 motor and loading machine speed transients were demonstrated for sigmoid reference track, for the drive system with PI and FL speed controllers for both control indexes.
Fig. 7. The drive system behaviour for $K_1$ control index: (a, b) PI controller, c, d) FL controller:
(a, c) $\omega_{\text{max}} = 1$, (b, d) $\omega_{\text{max}} = 0.5$

Rys. 7. Przebiegi zmiennych układu dwumasowego dla wskaźnika jakości sterowania $K_1$ liniowego (a,b) oraz rozmytego regulatora FL (c,d) dla zadanej prędkości układu (a, c) $\omega_{\text{max}} = 1$, (b, d) $\omega_{\text{max}} = 0.5$

Fig. 8. The drive system behaviour for $K_2$ control index: (a, b) PI controller, (c, d) FL controller:
(a, c) $\omega_{\text{max}} = 1$, (b, d) $\omega_{\text{max}} = 0.5$

Rys. 8. Przebiegi zmiennych układu dwumasowego dla wskaźnika jakości sterowania $K_2$ liniowego (a,b) oraz rozmytego regulatora FL (c,d) dla zadanej prędkości układu (a, c) $\omega_{\text{max}} = 1$, (b, d) $\omega_{\text{max}} = 0.5$
Tests were performed for nominal load torque of the drive system. In both cases the transients of the motor and loading machine speed were similar. For FL controller the system response was little bit faster and speed overshoots were slightly smaller, especially for the $K_2$ control index. The situation should be completely different when PI controller was adjusted according to the classical symmetry criterion, what is very popular in practice; in such case the behaviour of the drive system with FL controller would be much better.

7. THE INFLUENCE OF MECHANICAL PARAMETERS ON THE DRIVE SYSTEM DYNAMICS

The influence of mechanical parameters of the drive system, such as time constant of the elastic joint $T_c$ or mechanical time constant of the load machine $T_{M2}$ was also tested. In the following figures some transients of the system are presented.

![Transients of angular speed of loading machine](image)

Fig. 9. Transients of angular speed of loading machine $\omega_L$ for $K_I$ (a, b) and $K_2$ (c, d) control indexes for PI and FL controllers and changed load inertia: (a, c) for $\omega_{ref} = 1$, (b, d) for $\omega_{ref} = 0.1$
In Fig. 9 the case of significant change of inertia torque of the loading machine was demonstrated for two speed reference values and both control indexes used for controllers’ optimisation.

![Graphs showing speed transients](image)

As can be concluded from these figures concludes, that even for $K_1$ control index the FL controller ensure better damping of speed oscillations in whole speed range. This feature is multiplied for more nonlinear control index $K_2$ (fig.9 c, d) in both operating condition of the drive system: for speed reference changes as well as for load torque changes. It confirms that fl controller is more robust to changes of operation conditions or parameter changes of the drive system. In fig. 10 and 11 similar tests were demonstrated for the sigmoidal speed reference.
8. CONCLUSION

The results of simulation tests performed for the two-mass DC drive systems with two kinds of speed controllers were presented in the paper. The dynamic behaviour of the system with the classical PI controller and nonlinear FL controller were compared, especially for the low speed region operation and for changing parameters of the drive.

Both kinds of speed controllers were designed using the hybrid genetic-gradient algorithm for two different control indexes. It was proved that for more complicated and nonlinear control index the FL controller ensure much better dynamical properties of the two-mass drive system and is robust to drive system parameter changes or parameter identification errors. It was proved that the two-mass drive system with FL controller has better dynamic performances in case of nominal as well as for changed drive
parameters than classical PI controller.

APPENDIX

Main data of the drive system are as follows:
\[ T_{M1}=148\text{ms}, \ T_{M2}=148\text{ms}, \ T_c=1\text{ms}, \ K_t=7.4, \ T_e=1.4\text{ms}, \ h=0.01. \]

REFERENCES

PORÓWNANIE WŁAŚCIWOŚCI DYNAMICZNYCH UKŁADU DWUMASOWEGO Z REGULATOREM PI ORAZ ROZMYTYM W ZADANIU ŚLEDZENIA TRAJEKTORII

Artykuł dotyczy analizy właściwości dynamicznych napędu prądu stałego z połączeniem elastycznym i różnymi rodzajami regulatorów prędkości, w zadaniu śledzenia trajektorii. Rozważano kaskadową strukturę sterowania silnika z dwoma sprzężeniami zwrotnymi prędkościowymi. Porównano właściwości dynamiczne napędu z klasycznym regulatorem prędkości typu PI oraz z regulatorem rozmytym. Parametry obu regulatorów były optymalizowane przy zastosowaniu tych samych wskaźników jakości sterowania oraz algorytmu genetyczno-gradientowego. Przedstawiono wyniki badań symulacyjnych badanych układów.