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**BEHAVIOR OF THE SINGLE-PULSE OPERATION SWITCHED RELUCTANCE MOTORS**

The paper deals with analytic solution of currents and torque of Switched Reluctance Motor resulting in calculation of switching angles of the power transistors in supplying converter. The course of the inductance is replaced by a Fourier series.

1. INTRODUCTION

Switched reluctance motors are becoming to be an attractive replacement of the frequency controlled asynchronous motor in a wide range of applications. Due to its simple construction is the SRM considered a very reliable machine not requiring any maintenance.

Torque direction of the machine does not depend on the direction of the excitation current but it depend on the mutual position of the stator pole and rotor tooth. From this reason, the excitation can be unipolar, that considerably simplifies supply and control circuits.

![Geometry of reluctance machine](image)

**Fig. 1. Geometry of reluctance machine**

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We suppose a 6/4 motor, having 6 stator poles and 4 rotor teeth, as shown in Figure 1. It is assumed that the phases are magnetically independent and that the magnetic circuits is not saturated and there are not any losses.

Let’s assume the zero position of the phase 1–1′ is equal to the aligned rotor position, as shown in Figure 1. The unaligned position of rotor will by at the angle \( \frac{\pi}{N_r} \), where \( N_r \) is number of rotor teeth.

2. ELECTRIC MODEL

The analysis of the machine will by start from the electric model shown in the Figure 2. As the mutual inductance among the phases is neglectible, it is sufficient to consider the only one motor phase [1]. This one comprises ohmic resistance of the coil winding and induced voltage caused by change of the inductance and current.

\[
\begin{align*}
\frac{d\psi}{dt} & = R_i + \frac{d\psi}{dt} \\
\end{align*}
\]

where \( \psi \) is total magnetic flux of the coil.

Magnetizing curve is expressed by expression:

\[
\psi = L(\theta, i) \theta
\]

where \( L(\theta, i) \) is dynamic inductance of the phase.

It depends on the mutual position of the stator pole and rotor tooth. In the saturation zone it depends also on the value of excitation current.

After substitution into (1) on a receives:

\[
\begin{align*}
u & = R_i + L(\theta, i) \frac{di}{dt} + i \frac{dL(\theta, i)}{dt} \\
\end{align*}
\]

where \( i \frac{dL(\theta, i)}{dt} = e \) is the electromotive force of the phase.
Let’s suppose the change of electric variables is much quicker than change of mechanical variables. In quasi-steady state at $\omega = \text{cst.}$ can the time be substituted by the angle rotation through the rotor angle:

$$u = Ri + L(\theta, i)\omega \frac{di}{d\theta} + i\omega \frac{dL(\theta, i)}{d\theta}$$

(4)

In a motor with no magnetic saturation the magnetization curves would by straight lines. At any position the coenergy and the stored magnetic energy are equal, and are given by:

$$W = W' = \frac{1}{2} L(\theta)^2$$

(5)

where $L(\theta)$ is the inductance at a particular position.

In this case the instantaneous torque reduces to:

$$\gamma = \frac{dW}{d\theta} = \frac{1}{2} i^2 \frac{dL(\theta, i)}{d\theta}$$

(6)

where $W$ is the work performed during conversion cycle.

3. COURSE OF INDUCTANCE

The inductance curves $L(\theta)$ for non-saturable motor are of the form shown in Figure 3.

A Fourier series can express the curve of inductance:

$$L = L_0 + L_1 \cos \theta + L_2 \cos 2\theta + \ldots$$

(7)
For practical utilization it is sufficient to take only four or five first terms of (7). Let’s take for our calculation only three terms.

4. CALCULATION OF CURRENTS AND TORQUE

The current curve is described by the differential equation (4). After substituting from (7) into (4) we obtain:

\[ u = Ri - i\omega(L_1 \sin \theta + 2L_2 \sin 2\theta) + \omega(L_0 + L_1 \cos \theta + L_2 \cos 2\theta) \frac{d}{d\theta} \]

(8)

After its arrangement on receive:

\[ \begin{align*}
-\omega(L_1 \sin \theta + L_2 \sin 2\theta) & d\theta + \omega(L_0 + L_1 \cos \theta + L_2 \cos 2\theta) & di = 0
\end{align*} \]

(9)

The equation (9) presents the exact differential equation of the type:

\[ P(\theta, i) d\theta + Q(\theta, i) di = 0 \]

(10)

This equation has an analytical solution provided that:

\[ \frac{\partial P(\theta, i)}{\partial i} = \frac{\partial Q(\theta, i)}{d\theta} \]

(11)

where:

\[ \left( \frac{\partial}{\partial i} P \right) = R - \omega(L_1 \sin \theta + 2L_2 \sin 2\theta) \]

\[ \left( \frac{\partial}{\partial \theta} Q \right) = -\omega(L_1 \sin \theta + 2L_2 \sin 2\theta) \]

The condition is fulfilled at neglecting the excitation winding resistance \( R = 0 \). By this neglecting the equation (9) will by simplified:

\[ \begin{align*}
-\omega(L_1 \sin \theta + L_2 \sin 2\theta) & d\theta + \omega(L_0 + L_1 \cos \theta + L_2 \cos 2\theta) & di = 0
\end{align*} \]

A general solution of this exact differential equation has the form of:

\[ u\theta - i\omega(L_0 + L_1 \cos \theta + L_2 \cos 2\theta) = C \]

(12)

A general solution has this form for whatever number of terms of (7).
5. DYNAMIC OPERATION

The flux in the switched reluctance motor is not constant, but must be established from zero every stroke. In motoring operation the build-up is timed to coincide with the period when the rotor poles are approaching the stator poles of the phase.

Assuming that each phase is supplied by a circuit of the form of Figure 4.

Both transistors $T_1$, $T_2$ are switched on turn on angle $\theta_a$ and both are switched off at turn off angle $\theta_d$.

The integral constant $C$ can be calculated for interval $T_1$, $T_2$ on from the condition initial:

$$ i = 0 \quad \text{for} \quad \theta = \theta_a \quad \text{and} \quad u = U $$

By substituting into (12) and its arrangement we get the equation describing the excitation current curve in this interval:

$$ i = \frac{U(\theta - \theta_a)}{\omega(L_0 + L_1 \cos \theta + L_2 \cos 2\theta)} \quad (13) $$

For interval $T_1$, $T_2$ off the integral constant $C$ can be calculated from condition:

$$ i = I_d \quad \text{for} \quad \theta = \theta_d \quad \text{and} \quad u = -U $$

For this interval the current curve can be expressed as:

$$ i = I_d - \frac{U(\theta_d - \theta)}{\omega(L_0 + L_1 \cos \theta + L_2 \cos 2\theta)} \quad (14) $$

The expressions (13) and (14) are valid for any number of terms Fourier series of inductance.
1. CURVES OF CURRENTS AND TORQUE

For calculation of currents and torque, the three-phase reluctance machine was considered with the ratio equal to 6/4. Further parameters:
- supply voltage: \( U = 100 \text{ V} \);
- inductance in aligned position 1.2 H;
- inductance in unaligned position 0.2 H;
- motor speed \( \omega = 100 \text{ rad/s} \);
- turn on angle \( \theta_a = -3 \text{ rad electric} \);
- turn off angle \( \theta_d = -1.2 \text{ rad electric} \).

The course of inductance is expressed by:

\[
L = 0.6 + 0.5 \cos \theta + 0.1 \cos 2\theta + 0.01 \cos 4\theta
\]

The figure 5. shows the curves of the inductance, the supply voltage, the current and the torque of the phase as a function of rotor position.

7. CONCLUSION

The contribution shows an analytic calculation of currents and torque of SRM. At calculation, the influence of dissipation and local saturation of magnetic field is neglected.
The analytic solution enables to determine the optimal turn on and turn off angle of the power transistors supplying the motor.

REFERENCES

[1] ZÁSKALICKÝ P., FEDÁK V., Analytical Solution of Switching Angle for Switched Reluctance Motors; Proceeding of the 8th International Power Electronic & Motion Control Conference, s. 3.144–3.147, 8–10 September, 1998, Prague, Czech Republic.


WŁAŚCIWOŚCI JEDNOPULSOWO PRZEŁĄCZALNEGO SILNIKA RELUKTANCYJNEGO

Przedstawiono dla założonych kątów załączenia tranzystorów mocy w przekształtniku zasilającym analityczne zależności dla prądów i momentu w silniku reluktancyjnym przełączalnym (SRM). Przebiegi indukcyjności zastąpiono szeregami Fouriera.