The paper presents results of analysis of influence of chosen parameters of induction cage motor on the component \((1 - 2s)f_0\) in stator currents, which is commonly used for diagnosis of cage condition. The investigation concentrates on the influence of commonly available motor data such as pole-pair and rotor slot numbers, respectively, but also on relationships between resistances of cage bars and end rings. A reduced model of motor with a faulty cage is used, in which cage faults are represented by asymmetry coefficients of a cage. The main intention of the paper is to provide engineers with simple estimates of effects due to cage faults for various motors given limited information.

1. INTRODUCTION

Nowadays diagnosis of induction motor cage faults bases on the Motor Current Signature Analysis (MCSA). Many papers [1]–[7] are devoted to that problems. Prediction of effects in stator currents due to cage faults is provided engaging circuital or field models of Induction Motors (IM) that are more or less complicated. They required detailed information on design data of a motor, which usually are not available for engineers in the industry. Therefore, it is requested to estimate effects of cage fault using only basic data of a motor. This paper is an attempt to answer to that problem given limiting necessary information to the pole-pair number “p” and the rotor slot number “N”.

Commonly, the component with the frequency \((1 - 2s)f_0\) in stator phase currents is recognized as a basic fundamental effect of cage faults. The effect can be predicted by a basic model of induction motors accounting for the cage asymmetry. One such model was presented in [8]. It operates with symmetrical components of stator and rotor quantities, and it is a simple extension of the “classical” IM model. The cage
asymmetry is represented in the model by the asymmetry factors \( k_{as} \) and \( k_s \). In the case of a “healthy” cage those factors are equal to zero.

2. ESTIMATION OF EFFECTS IN CURRENTS DUE TO CAGE FAULTS

Various studies [9], [11] presented a dynamic model of IM considering the main MFF harmonic only and a cage asymmetry. When supply voltages are balanced and steady-states are considered that equation are reduced to four algebraic equations of the form

\[
\begin{bmatrix}
U^1_s \\
0 \\
0 \\
0
\end{bmatrix} = j
\begin{bmatrix}
X_{\sigma S} + X_\mu & 0 & X_\mu & 0 \\
0 & X_{\sigma S} + X_\mu & 0 & X_\mu \\
X_\mu & 0 & X'_{\sigma r} + X_\mu & 0 \\
0 & X_\mu & 0 & X'_{\sigma r} + X_\mu
\end{bmatrix}
\begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix} + 
\begin{bmatrix}
R_s & 0 & 0 & 0 \\
0 & \frac{R_s}{2s-1} & 0 & 0 \\
0 & 0 & \frac{R'_{r/s}}{s}(1+k_s) & \frac{R'_{r/s}}{s} k_{as} e^{-j\alpha} \\
0 & 0 & \frac{R'_{r/s}}{s} k_{as} e^{j\alpha} & \frac{R'_{r/s}}{s} (1+k_s)
\end{bmatrix}
\begin{bmatrix}
I^1_s \\
I^2_s \\
I'^r_s \\
I'^N_s
\end{bmatrix}
\]

where \( I^1_s \) denotes a value of the first symmetrical components \( i^1_s(t) = I^1_s \cdot e^{j\Omega_0 \cdot t} \) of the supply frequency \( \Omega_0 \) and \( I^2_s \) a value of the second component \( i^2_s(t) = I^2_s \cdot e^{-j(1-2s)\Omega_0 \cdot t} \) with the slip dependent frequency \( (1-2s) \cdot \Omega_0 \). Those equations could be represented by the equivalent scheme presented in Fig. 1.

![Fig. 1. Equivalent scheme of induction motors with faulty cage](image)

From that equivalent scheme a relationship between the main current component \( I^1_s \) and the component \( I^2_s \), generated by a cage fault, can be found. In [11] it has been
shown that at normal operating conditions of a motor, i.e. for small slips, the magnetizing reactance $X_\mu$ and the leakage reactances $X_{\sigma,s}$, $X_{\sigma,r}$ can be omitted, respectively, and neglecting the resistance $R_s$, the ratio between $I_s^2$ and $I_s^1$ can be expressed by the cage asymmetry factor $k_{as}$

$$k_{1-2s} = \frac{|I_s^2|}{|I_s^1|} = k_{as}$$ (2)

In Figure 2 the values of the ratio factor $k_{1-2s}$ are shown versus slip at different assumptions for equivalent scheme parameters. The curve “a” is obtained from a full scheme, the curve “b” was developed when omitting the magnetizing reactance $X_\mu$. The curve “c” is obtained when leakage reactances are omitted as well, i.e. for a purely resistive scheme. The dashed line (curve “d”) shows the value $k_{as}$. Those curves were calculated for a motor with the following rated data: $P_n = 2000$ kW, $n_n = 2980$ rpm, $U_s = 6$ kV, $I_n = 227$ A, $p = 1$, $N = 36$.

![Fig. 2. The ratio $k_{1-2s}$ versus slip at different simplification of the equivalent scheme in Fig. 1 [11]](image)

It is evident that the ratio $k_{1-2s}$ is very close to asymmetry factor $k_{as}$ for near rated slips.

3. ESTIMATION OF ASYMMETRY FACTOR BY POLE-PAIR AND ROTOR SLOT NUMBERS

To determine the asymmetry factor $k_{as}$, the resistances of each cage elements should be known [8]–[11], i.e. resistances of cage bars $R_b$ and end-ring segments $R_e$. 
Usually, cage faults are modelled by increasing the resistance of a faulty element up to 20 times the nominal value. The asymmetry factor $k_{as}$ determines a resistive coupling between equivalent schemes for the first and the second symmetrical components, which are connected back-to-back (see Fig. 1) in comparison to the value of the rotor resistance in the equivalent scheme for the symmetrical case

$$R'_p = (v)^2 \cdot 2 \cdot \left( R_e + 2 \cdot R_b \cdot \sin^2 \left( \frac{p \cdot \pi}{N} \right) \right)$$

(3)

where “$p$” is a pole-pair number and “$N$” is a number of bars in a cage. The resistances $R_b$ and segments $R_e$ can be related using design data of a motor. Knowing the relationship between currents in bars and end ring segments $I_b = 2 \cdot I_e \cdot \sin(\alpha/2)$, where $\alpha = (2 \cdot \pi \cdot p)/N$, and estimating the relationship between the bar length “$L$” and the diameter of end rings “$D$” as $L = (\lambda \cdot \pi \cdot D)/2 \cdot p$ as well as using the machine ratio $\lambda = L/\tau$, where $\tau$ is the pole pitch, the relationship between the resistances $R_b$ and $R_e$ is done by the formula

$$R_e = \frac{4 \cdot \pi}{\lambda} \cdot \left( \frac{p}{N} \right)^2 \cdot \sin \left( \frac{\pi \cdot p}{N} \right) \cdot \frac{j_e \cdot \gamma_b}{j_b \cdot \gamma_e} \cdot \frac{1}{k_{skin}} \cdot R_b$$

(4)

where: $j_b, j_e$ are current densities in bars and end-ring segments, $\gamma_b, \gamma_e$ are respective conductivities and $k_{skin}$ is a factor for determining skin effects in a bar.

Based on this relation the asymmetry factor $k_{as}$ can be estimated by using a pole-pair number and a number of cage slots only and, as a consequence, the ratio $k_{1-2s}$. The value of the factor $k_{as}$ depends on the fault type. This paper shows that it is an essential case when one bar is broken because effects of multi-bars faults can be estimated by that elementary fault.

### 4. INFLUENCE OF END-RING SEGMENT RESISTANCE ON ASYMMETRY FACTOR

From (2) it follows that relationships between resistances $R_b$ and $R_e$ depend on the conductivity of a material used for cage and current densities assumed for bars and end-rings. Usually, materials used for end-rings and bars are the same, so the ratio $\gamma_b/\gamma_e$ is equal to one. For small slips the skin effect can be neglected as well. So, an influence of end-ring geometry on the asymmetry factor can be studied by changing the ratio $j_e/j_b$.

Results of calculations of the asymmetry factor $k_{as}$ for one broken bar are performed given the following assumptions: $\lambda = 1.5, k_{skin} = 1.0$ and $\gamma_b/\gamma_e = 1.0$ – see Fig. 2.
There are four graphs in this figure, when the quotient $j_e/j_b$ is changing. The graph “a” shows the factor $k_{as}$ versus the value “$1/N$” for different pole-pair numbers (from $p = 1$ till $p = 5$) for $j_e/j_b = 1.0$. The curve “b” shows the same data for $j_e/j_b = 2.0$ and graph “c” for the ratio $j_e/j_b = 0.5$. In the graph ‘d’ the resistance of end-ring segments is assumed to be zero ($R_e = 0$). Such assumption is sometimes used when analyzing the cage asymmetry. It can be seen that for real values of the quotient $j_e/j_b$ the asymmetry factor $k_{as}$, and also the ratio $k_{1-2s}$, depends on the number of slots in a cage but also on the pole-pair number. It can be stated that end-rings can be omitted for higher slots numbers and then $k_{1-2s} \approx 1/N$.

Fig. 3. Influence of end-ring segment resistance on the asymmetry factor for different pole-pair numbers
5. ESTIMATION OF RATIO $k_{1-2s}$ FOR MULTI-BAR FAULTS

In the case of multi-bar faults the ratio $k_{1-2s}$ can be estimated by the asymmetry factor for one broken bar. In literature the relationship $k_{1-2s} \approx n/N$ is often used, where “n” is a number of broken bars. However, this relationship is not valid even for simple multi-bar faults. A far more exact estimation of the ratio $k_{1-2s}$ for multi-bar faults can be obtained by summing geometrically the components $(1 - 2s)f_0$ generated by individual broken bars as it is explained in Fig. 4 for the case of two successive broken bars.

![Fig. 4. Estimation of effects due to two adjacent broken bars](image)

It follows that for $n$ sequent broken bars the ratio $k_{1-2s}$ can be estimated by the following formula

$$k_{1-2s} = k_{as} = \frac{\sin \left( n \cdot \frac{\gamma}{2} \right)}{n \cdot \sin \left( \frac{\gamma}{2} \right)} \cdot k_{1,as}$$

(5)

where $k_{1,as}$ is the asymmetry factor for one broken bar and $\gamma = 2 \cdot \alpha = 2 \cdot (2 \cdot \pi \cdot p)/N.$ That estimation is much more meaningful than $k_{1-2s} \approx n / N.$ The changes of ratio $k_{1-2s} = k_{as}$ for neighbour broken bars versus “1/N” at different pole-pair numbers are shown in Fig. 5: graph “a” for two bars and graph “b” for three bars. All curves have been obtained for $\lambda = 1.5$, $k_{skin} = 1.0$, $\gamma_b/\gamma_e = 1.0$ and $j_e/j_b = 1.0$.

![Graph a](image)

![Graph b](image)

Fig. 5. Asymmetry factor at 2 and 3 neighbour bars broken for different pole-pair numbers
Those figures illustrate difficulties in diagnosing multi-bars faults. The geometrical sum of \((1 - 2s)f_0\) components generated at individual broken bars can take an arbitrary value and can even disappear at small numbers of cage slots and bigger pole-pair numbers.

![Evolution of \(k_{as}\)](image)

Fig. 6. Asymmetry factor for versus positions of a pair of broken bars

That effect can be observed also in Fig. 6, in which changes of the asymmetry factor are shown for two bars are broken located at various positions. In that figure the positions of a broken bar pair are denoted as: “1–2”, “1–3”, “1–4” etc. The calculations were performed for a motor with \(N = 36\) bars for pole-pair numbers \(p = 1\), \(p = 2\) and \(p = 3\), and given \(\lambda = 1.5\), \(k_{skin} = 1.0\), \(\gamma_b\gamma_e = 1.0\) and \(j_e/j_b = 1.0\). Also, it should be noted that for each pole-pair number there are pairs for which asymmetry factor is close to zero, meaning that the effect of cage asymmetry is not observed in stator currents. Therefore, it is rather evident that the ratio \(k_{1-2s}\) is not uniquely related to a multi-bar fault type. The final conclusion of the presented analysis is that the diagnostics of cage fault based on \((1 - 2s)f_0\) components in stator currents could be unsatisfactory. In practice, diagnostic systems should attempt to identify the onset of cage damage because when multi-bar fault arise the analysis could be unsatisfactory.

6. CONCLUSIONS

In the paper a simple analysis of effects in stator currents due to cage faults is presented. It has been shown how to estimate effects of multi-bar cage faults based on the effect of one broken bar. The list of parameters required for that estimation has been
limited to the number of pole-pairs and the cage slot number. The influence of end-
ring design data on effects due to cage damage has been also discussed. Estimation
presented in the paper can be useful for engineers for cage diagnostics.

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