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A TUNING CRITERION FOR DISCRETE CURRENT COMPENSATORS OF ELECTRIC DRIVES

Analytical selection of the proportional and the integral gains for discrete PI-regulators dedicated to current control of electric motors is presented in this paper. A step-by-step tuning procedure starts with basic formulas in the Laplace domain. Further, more advanced, path leads to successful selection of the gains according to desired closed loop frequency bandwidth. The advanced steps walk through the $Z$-domain combined with the $\hat{w}$-transformation allowing use of well known selection techniques from the continuous domain. All the considerations are based on assumption that the control object is linear.

1. INTRODUCTION

Proportional-integral (PI) control is commonly used in the most inner control loops of the field oriented vector control schemes for electric drives [1, 2]. This control layer is responsible for the motor phase currents and therefore for the electromagnetic torque too. In other words, stability (in specified operating conditions) and dynamics (at given PWM switching frequency) of the loops are particularly important for the torque control of an electric motor.

The stability and dynamics are mainly determined by values of proportional and integral gains at given system operating conditions. In case of linear time-invariant systems the gains are of fixed values for all the system operating points. In continuous domain there is a number of methods which could be used for the gains selection [3, 4].

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In case of a real time embedded system implementation the current control algorithm runs in a discrete manner. This due to the way of signal sampling and processing in microprocessors or DSPs and due to finite frequency of the PWM.

The discrete nature of digital control systems calls for proper addressing while selecting values of the proportional and the integral gains [5] and this is the subject of this paper.

2. CONTROL BASICS

Basic structure of a closed-loop control system in the Laplace domain can be drawn as in fig. 1. Such a well known representation is a starting point for advanced considerations regarding tuning of digital current regulators – considerations basing purely on general control theory.

\[
\frac{Y(s)}{R(s)} = \frac{C(s)P(s)}{1 + C(s)P(s)} \quad (1)
\]

where the \( C(s) \) stands for the regulator transfer function and the \( P(s) \) for a plant under control. The \( R(s) \) is the reference signal, \( E(s) \) is the error, \( U(s) \) is the input signal of the plant (the supply) and finally the \( Y(s) \) is the plant output (controlled quantity).

Fig. 1. Simplified block diagram of a closed loop control system in the Laplace domain

Fig. 2. Simplified block diagram of a closed loop control system in the Z-domain
In case of a sampled data control system signals are read, computed and set with a certain frequency. The frequency can be fixed or varying. In case of electric drives most often it is a constant PWM frequency and its aliquot parts.

For purpose of control analysis a sampled data closed loop control system can be represented in the \( z \)-domain as shown in fig. 2, with the transfer function (2).

\[
\frac{Y(z)}{R(z)} = \frac{C(z)P(z)}{1 + C(z)P(z)}
\]  

(2)

Difference between (1) and (2) rests inside of their plant \( (P) \) and regulator \( (C) \) representation. In case of a PI regulator in the Laplace domain the \( C(s) \) can be written as following:

\[
C(s) = K_p + K_i \frac{1}{s}
\]  

(3)

where the \( K_p \) is the proportional gain, the \( K_i \) is the integral gain. In the \( z \)-domain, assuming the backward Euler integration method, the equation is as following:

\[
C(z) = K_p + K_i \frac{T \cdot z}{z - 1}
\]  

(4)

where the \( T \) is the sampling time. The equation (4) can also be written for accumulation based backward Euler integration method, (5), – this form for simplicity reason will be used in the paper.

\[
C(z) = K_p + K_i \frac{z}{z - 1}
\]  

(5)

The plant transfer function will be discussed in details further on.

3. FIELD ORIENTED CONTROL

One of the most popular solutions for control of modern electric drives is the field oriented vector control scheme. The scheme relies on cascaded control structure with separated control layers for position (the top level), velocity and current/torque (the most inner loop). An example of the most inner control loops, in case of permanent magnets synchronous motors (PMSM), is shown in fig. 3. The reference currents \( (i_{qref}, i_{dref}) \) are set in the outer control loops.
In case of the current control loops gains of PI compensators must be selected taking into account resistance and inductance of electric motor windings. In simplified form a winding could be represented as shown in fig. 4. Such an equivalent circuit is a starting point for the proportional and the integral gains selection in an analytic way.

4. SELECTION OF THE CURRENT CONTROLLER GAINS

In terms of control analysis voltage equation (6), in the continuous time domain, is the starting point. As next its Laplace domain representation is introduced, (7). Basing on the (7) a plant transfer function, current to voltage, is described by equation (8).

![Motor windings representation for the PI gains selection](image)

\[
v(t) = R i(t) + L \frac{d i(t)}{dt}
\]

(6)

\[
V(s) = RI(s) + sLI(s)
\]

(7)
\[
\frac{I(s)}{V(s)} = P(s) = \frac{1/R}{s \frac{L}{R} + 1} \tag{8}
\]

The equation (8) can be simplified with substitution of \( \frac{L}{R} = \tau \) – the circuit time constant. Equation (8) does not reflect nature of sampled data control system. A simple way of doing it is an introduction of the Zero-order hold expressed by the following equation [5]:

\[
H_0(s) = \frac{1 - e^{-s\tau}}{s} \tag{9}
\]

The equation (9) stands for linear model of an analogue to digital converter used for current sampling. Combining (8) and (9) leads to the plant transfer function, (10), ready to be used in further steps towards the \( K_P \) and the \( K_I \) selection.

\[
P(s) = (1 - e^{-s\tau}) \frac{1/R}{s(s \frac{L}{R} + 1)} \tag{10}
\]

At this stage a decision must be made whether further work will be performed in the Laplace domain or in the \( Z \)-domain. The \( Z \)-domain seems to be a natural and well proven path for discrete systems [6]. The equation (10) can be represented in the \( Z \)-domain as following:

\[
P(z) = (1 - z^{-1}) \frac{1}{R} \left( \frac{z}{z - 1} - \frac{z}{z - e^{-\frac{\tau}{T}}} \right) = \frac{1 - e^{-\frac{\tau}{T}}}{R} \frac{1}{z - e^{-\frac{\tau}{T}}} \tag{11}
\]

Introducing simplifications in equation (11),

\[
k_{pg} = \frac{1}{R} (1 - e^{-\frac{\tau}{T}}) \tag{12}
\]

\[
k_{pp} = e^\frac{\tau}{T} \tag{13}
\]

where the \( k_{pg} \) is the plant gain and the \( k_{pp} \) is the plant pole, a simple plant transfer function can be written as:
\[ P(z) = \frac{k_{pg}}{z - k_{pp}} \]  

(14)

Assuming that current sampling delay can be compensated by a predictive current observer [7], and that the PWM unit transfer function is equal to one – a simple block diagram can be created, see fig. 5. The diagram may be used for further examination of the current control aspects.

![Simplified block diagram of an electric motor current control structure](image)

Fig. 5. Simplified block diagram of an electric motor current control structure

The open loop transfer function related to fig. 5 can be written as following:

\[ T_{ol}(z) = (K_P + K_I) \frac{z}{z-1} \frac{k_{pg}}{z - k_{pp}} = \gamma \frac{(z - \frac{K_P}{K_P + K_I})}{(z-1)(z-k_{pp})} \]  

(15)

where:

\[ \gamma = k_{pg} (K_P + K_I) \]  

(16)

It can be seen in (15) that the controller creates a pole at \( z=1 \) and a zero in function of the \( K_P \) and \( K_I \). In this circumstance the first criterion for the controller gains can be stated as: place the controller zero in such a way that the open loop plant pole will be cancelled, (17).

\[ k_{pp} = \frac{K_P}{K_P + K_I} \]  

(17)

Basing on (17) the (15) can be rewritten in simplified form as:

\[ T_{ol}(z) = \frac{\gamma}{(z-1)} \]  

(18)

and in result the closed loop transfer function is:

\[ T_{cl}(z) = \frac{\gamma}{(z-1 + \gamma)} \]  

(19)
Going further the \( \hat{w} \)-transform theory, \([5]\), can be applied to (19). Substituting the “\( z \)” by

\[
   z = \frac{(1 + wT)}{2} \quad \frac{(1 - wT)}{2}
\]

leads to

\[
   T_{cl}(w) = \frac{\gamma(1 - \frac{wT}{2})}{wT + \gamma(1 - \frac{wT}{2})} = -\frac{\gamma}{2 - \gamma} \frac{(w - \frac{2}{T})}{(w + \frac{2\gamma}{T}(2 - \gamma))}
\]

Further steps can comply with the continuous system design rules. It is allowed to make the well known substitution \( w = j\omega \). This leads to:

\[
   T_{cl}(j\omega) = -\frac{\gamma}{2 - \gamma} \frac{(j\omega - \frac{2}{T})}{(j\omega + \frac{2\gamma}{T}(2 - \gamma))}
\]

Looking at (22) one can see that: 1) there is a right hand plane zero at \( 2/T \) driven by the sampled nature of the system, 2) the closed loop bandwidth may be set depending on the controller gains. Speaking in terms of Bode plot the absolute value of the amplitude from (22) can be compared against the \(-3dB\) point, where the signal is down by \( \frac{1}{\sqrt{2}} \).

\[
   \frac{1}{\sqrt{2}} = \frac{\gamma}{2 - \gamma} \frac{\sqrt{\omega^2 + \left(\frac{2}{T}\right)^2}}{\sqrt{\omega^2 + \left(\frac{2\gamma}{T(2 - \gamma)}\right)^2}}
\]

At this stage the second criterion for the controller gains can be derived from combination of the equation (23) with (16).

\[
   K_p + K_I = \frac{2}{k_{pg}} \frac{1}{1 + \sqrt{2 + \left(\frac{2}{T\omega}\right)^2}}
\]
Relation between general frequency $\omega$ and the closed loop bandwidth, $f_{3dB}$ or $\omega_{3dB}$, can be expressed as following, [5]:

$$\omega = \frac{2}{T} \tan \left( \frac{\omega_{3dB} T}{2} \right) = \frac{2}{T} \tan \left( \pi f_{3dB} T \right)$$  \hspace{1cm} (25)

At this stage the $K_P$ can be calculated from (17) in combination with (24) and (25).

$$K_P = \frac{k_{pp}}{k_{pg}} \frac{2}{1 + \sqrt{2 + \left( \frac{1}{\tan(\pi f_{3dB} T)} \right)^2}}$$ \hspace{1cm} (26)

With the given $K_P$, (26), the $K_I$ can be calculated from (24) and (25).

$$K_I = \frac{2}{k_{pg}} \frac{1}{1 + \sqrt{2 + \left( \frac{1}{\tan(\pi f_{3dB} T)} \right)^2}} (1 - k_{pp})$$ \hspace{1cm} (27)

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Fig. 6. Closed loop Bode plots (magnitude only) obtained with different sets of PI gains for different target bandwidths for $R = 3.35 \Omega$ and $L = 7.2$ mH: $f_{3dB} = 750$ Hz ($K_P = 26.1$ V/A, $K_I = 1.24$ V/As); $f_{3dB} = 1.5$ kHz: ($K_P = 41.15$ V/A, $K_I = 1.96$ V/As); $f_{3dB} = 3$ kHz: ($K_P = 54.32$ V/A, $K_I = 2.59$ V/As)
The gains calculated according to (26) and (27) have been verified with Bode plots (magnitude), fig. 6, and step responses, fig. 7, in Matlab environment – this with current control structure as per fig. 2. The winding parameters used have been taken from a PMSM catalogue – resistance $R = 3.35 \, \Omega$, and inductance $L = 7.2 \, \text{mH}$. The switching frequency has been set to 10 kHz, $T = 100 \, \mu\text{s}$. Results have been combined for different controller bandwidths: 750 Hz, 1.5 kHz and 3 kHz.

5. CONCLUSIONS

The paper delivers formulas for discrete PI regulators tuning according to the desired closed loop control bandwidth. Complete derivation path of the formulas has been shown and discussed. The work assumes that the control object parameters are a given – in case of electric motors resistance and inductance of windings. Additionally it has been assumed that the control object is linear. In the reality nonlinearities coming from material properties may call for proper addressing – this through e.g. gain adaptation to changing operating points.

Correctness of the formulas has been verified in simulations.

REFERENCES