AN INFLUENCE OF SUPPLY VOLTAGE FREQUENCY ON DYNAMIC PERFORMANCE OF A SINGLE-PHASE CAPACITOR INDUCTION MOTOR

The paper presents a modeling mathematical tool for prediction of characteristics and simulating results of dynamic operation of the single-phase capacitor induction motor for different values of the capacitor capacitance and for two widely used values of supply voltage frequency at no-load and rated load conditions. Developed mathematical model of the capacitor induction motor was implemented for calculation using Matlab/Simulink software. The simulation shown the usefulness of the model for the process of designing single-phase induction motors.

1. INTRODUCTION

Single-phase capacitor induction motors are commonly used as a drive for low-power fans, pumps and compressors. Generally, they are induction machines of the symmetrical rotor cage and two non-symmetrical stator windings (the main winding and auxiliary winding with starting or running capacitor) supplied with the same sinusoidal voltage source. In modelling performance characteristics of the induction motors circuit models of lumped parameters are still often used due to their simplicity and fast computation. At first approximation of the mathematical models linearity of magnetic circuit is assumed and slotting of the stator and rotor are neglected [1, 3–4]. These models may be fully useful for searching of better operating parameters or showing direction of the searching in design optimization.

In the paper a mathematical model of the single-phase capacitor induction motor in the stationary $dq$ system is described which was implemented for simulation of the motor’s start-up at no-load and application of nominal load torque after starting from no-load operation. The present paper deals with simulations of dynamic characteristics

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of a tested capacitor induction motor supplied from sinusoidal voltage source of 230V at frequency of 50 and 60 Hz for different values of capacitor capacitance placed in the auxiliary stator winding. It should be mentioned that mains electricity in Europe, Australia, Oceania, most regions of Africa and Asia and few countries in South America use the voltage of 220–240 V, 50 Hz. The voltage of 220–240 V, 60 Hz is used in South Korea, Philippines, Peru, some regions of Brazil and in some smaller countries of Central America, while most of the North American countries use 110–127 V, 60 Hz.

2. DESCRIPTION OF THE TESTED MOTOR AND MATHEMATICAL MODEL

The low power, single-phase capacitor induction motor was used for computational analysis. The motor has two stator windings: the main (M) and the auxiliary (A), of which magnetic axes are displaced by about 90°. The running capacitor (C) is plugged in series with the auxiliary winding. The rotor has squirrel cage of 11 bars, that are distributed uniformly along circumference. Table 1 lists the ratings and structural data of the tested motor.

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Rated power</td>
<td>0.09 kW</td>
</tr>
<tr>
<td>Rated voltage</td>
<td>230 V</td>
</tr>
<tr>
<td>Rated current</td>
<td>0.9 A</td>
</tr>
<tr>
<td>Rated speed</td>
<td>2840 rev/min</td>
</tr>
<tr>
<td>Efficiency</td>
<td>0.55</td>
</tr>
<tr>
<td>Power factor</td>
<td>0.9</td>
</tr>
<tr>
<td>Frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Torque ratio</td>
<td>1.5</td>
</tr>
<tr>
<td>Number of stator windings</td>
<td>2</td>
</tr>
<tr>
<td>Running capacitor capacitance</td>
<td>3μF</td>
</tr>
<tr>
<td>Connection of stator windings</td>
<td>parallel</td>
</tr>
<tr>
<td>Number of poles: main/auxiliary winding</td>
<td>2/2</td>
</tr>
<tr>
<td>Winding layout: main/auxiliary winding</td>
<td>single layer</td>
</tr>
<tr>
<td>Rotor winding</td>
<td>squirrel cage</td>
</tr>
<tr>
<td>Number of slots: stator/rotor</td>
<td>18/11</td>
</tr>
<tr>
<td>Lamination material-generator sheet</td>
<td>M 600-50A</td>
</tr>
<tr>
<td>Laminated core length</td>
<td>32 mm</td>
</tr>
</tbody>
</table>

Since the axes of the both stator windings are orthogonal, the $dq$ axes of the model may be aligned with axes of the windings, as illustrated in Fig. 1. The squirrel cage rotor may be represented as a pair of symmetrical, short-circuited coils [2].
The circuit model of the capacitor induction motor can be described by set of differential equations, written as follows [3–5]:

\[
\begin{align*}
\frac{di_{dM}}{dt} &= \frac{R_s L_{rr}}{L_{\Sigma}} i_{dM} + \frac{L_{m}^2}{L_{\Sigma}} \omega_r i_{qA} - \frac{L_m L_{rr}}{L_{\Sigma}} i_{dr} + \frac{L_m L_{rr}}{L_{\Sigma}} \omega_r i_{qr} - \frac{L_{rr}}{L_{\Sigma}} u_{dM} \\
\frac{di_{qA}}{dt} &= -\frac{L_m^2}{L_{\Sigma}} \omega_r i_{dM} + \frac{R_s L_{rr}}{L_{\Sigma}} - \frac{L_m L_{rr}}{L_{\Sigma}} \omega_r i_{dr} - \frac{L_m L_{rr}}{L_{\Sigma}} \omega_r i_{qr} - \frac{L_{rr}}{L_{\Sigma}} u_{qA} + \frac{L_{rr}}{L_{\Sigma}} u_{qC} \\
\frac{di_{dr}}{dt} &= -\frac{R_s L_m}{L_{\Sigma}} i_{dM} - \frac{L_m L_{ss}}{L_{\Sigma}} \omega_r i_{qA} + \frac{R_s L_{ss}}{L_{\Sigma}} i_{dr} - \frac{L_m^2}{L_{\Sigma}} \omega_r i_{qr} + \frac{L_m}{L_{\Sigma}} u_{dM} \\
\frac{di_{qr}}{dt} &= \frac{L_m L_{ss}}{L_{\Sigma}} \omega_r i_{dM} - \frac{R_s L_m}{L_{\Sigma}} i_{qA} + \frac{L_m^2}{L_{\Sigma}} \omega_r i_{dr} + \frac{R_s L_{ss}}{L_{\Sigma}} i_{qr} + \frac{L_m}{L_{\Sigma}} u_{qA} - \frac{L_m}{L_{\Sigma}} u_{qC} \\
\frac{du_{qC}}{dt} &= \frac{1}{C} \iota_{qA} \\
\frac{d\omega_r}{dt} &= \frac{p^2}{J} L_m (\iota_{dM} i_{qA} - \iota_{qA} i_{dr}) - \frac{D_f}{J} \omega_r - \frac{p}{J} T_L 
\end{align*}
\]  

where:

\[
L_{\Sigma} = L_m^2 - L_{rr} L_{ss}, \quad R_s = R_m = R_f', \quad L_{ss} = L_{sM} = L_{sA}'
\]

and \(i_{dM}, i_{qA}\) are the direct- and quadrature-components of stator currents, \(i_{dr}, i_{qr}\) are the direct- and quadrature-components of rotor currents, \(u_{dM}, u_{qA}\) are the direct- and quadrature-components of stator voltages, \(u_{qC}\) is the voltage across the running capacitor, \(R_s\) is the resistance of stator windings, \(R_r\) is the resistance of rotor windings,
$L_{ss}$ is the self-inductance of stator windings, $L_{rr}$ is the self-inductance of rotor windings, $L_m$ is the stator magnetizing inductance, $p$ is the number of pole-pairs, $D_f$ is the viscous friction coefficient, $J$ is the moment of inertia, $T_L$ is the load torque, $\omega_r$ is the electrical angular velocity of the rotor.

3. DYNAMIC CHARACTERISTICS

The simulation results in this section are obtained for the single-phase induction motor described in paragraph 2, supplied with sinusoidal voltage sources of 230 V, 50 Hz and 60 Hz, at different capacitor capacitances, for free acceleration of the motor ($T_L = 0$) and applying the rated load ($T_L = T_N = 0.3 \text{ N\cdot m}$) after 0.7 s. The simulations were performed at the assumption, that only moment of inertia of the motor $J = 0.00007 \text{ kg\cdot m}^2$ and the viscous friction coefficient $D_f = 0.00005 \text{ N\cdot m\cdot s}$ were taken into account. The motor starting waveforms obtained for capacitor capacitance of 3 µF are shown in figures 2–4.

![Fig. 2. The electromagnetic torque $T_e(t)$ for $C = 3 \mu\text{F}$ and a) $f = 50 \text{ Hz}$, b) $f = 60 \text{ Hz}$](image)

![Fig. 3. The angular velocity $\omega_r(t)$ for $C = 3 \mu\text{F}$ and a) $f = 50 \text{ Hz}$, b) $f = 60 \text{ Hz}$](image)
It can be noticed, that when the motor is supplied with voltage of frequency 60 Hz, the rotor rotates with higher no-load and rated speed. The motor’s magnetic field rotates with synchronous speed about 377 rad/s, compared to about 314 rad/s for the line frequency 50 Hz. In addition to the average angular velocity, the line frequency influences also the shape of the rotating magnetic field in the machine’s air-gap. Oscillations of the electromagnetic torque (a result of an elliptical field) have noticeably lower amplitude at 60 Hz than at 50 Hz, especially when the motor operates under rated load conditions. With the increase of the voltage frequency, impedances of windings and running capacitor suitably change, currents in main and auxiliary windings have similar amplitudes and the rotating magnetic field is close to a circular field. Electromagnetic torque directly influences the waveform of the angular velocity – it’s oscillations under rated load conditions for 60 Hz have several times lower amplitude than for 50 Hz and they are practically negligible. The no-load current amplitude for 60 Hz is also noticeably smaller than for 50 Hz.

The simulation results for capacitor capacitances other than 3 µF are shown in figures 5–10.
Fig. 6. The angular velocity $\omega(t)$ for $C = 4 \ \mu F$ and a) $f = 50 \ \text{Hz}$, b) $f = 60 \ \text{Hz}$

Fig. 7. The intake current $i(t)$ for $C = 4 \ \mu F$ and a) $f = 50 \ \text{Hz}$, b) $f = 60 \ \text{Hz}$

Fig. 8. The electromagnetic torque $T_e(t)$ for $C = 6 \ \mu F$ and a) $f = 50 \ \text{Hz}$, b) $f = 60 \ \text{Hz}$
Fig. 9. The angular velocity $\omega_r(t)$ for $C = 6 \, \mu F$ and a) $f = 50 \, Hz$, b) $f = 60 \, Hz$

Fig. 10. The intake current $I(t)$ for $C = 6 \, \mu F$ and a) $f = 50 \, Hz$, b) $f = 60 \, Hz$

It can be observed, that when the running capacitor capacitance is increased to $4 \, \mu F$, the amplitude of the electromagnetic torque’s oscillation decreases for the motor supplied with voltage of frequency $50 \, Hz$. The start-up time also slightly decreases. However, for the line frequency of $60 \, Hz$, the amplitude of aforementioned oscillations increases, compared to $3 \, \mu F$, because the capacitor capacitance and line frequency influence on the phase difference between the main and auxiliary windings currents and also on the amplitudes of these currents. Further increase of the capacitance results in increase of amplitude of the electromagnetic torque oscillations, and consequently, the amplitude of the angular velocity oscillations.

4. CONCLUSIONS

In the paper, the dynamic behaviour of the single-phase capacitor induction motor supplied with a voltage of frequency of $50$ and $60 \, Hz$ was presented. The simulation
results showed, that for $C = 3 \, \mu F$, the line frequency of 60 Hz gives better results, as far as the oscillations of electromagnetic torque and angular velocity are concerned. Running capacitor capacitance of 4 $\mu F$ yields better results for 50 Hz since for 60 Hz, the torque’s oscillations have higher amplitude. It should be mentioned, that the tested motor was designed for operation with line frequency of 50 Hz. The supply voltage frequency also affects steady-state characteristics – when the motor is supplied with voltage of 60 Hz, the breakdown torque is lower than for 50 Hz [2].

The presented mathematical model of the single-phase capacitor induction motor may be used as an efficient mathematical tool for analytical prediction of performance characteristics of the motor already before its manufacturing.

REFERENCES