An FDC (Forced Dynamic Control) based speed control structure suitable for electric drives with significant flexibility in the coupling is proposed. The design methodology incorporates pole assignment to achieve a specified dynamic response to the reference input. The control structure embodies counteraction of the load torque acting on the load mass to minimize its effect on the load side speed. This also gives the system robustness against parametric uncertainties. The simulation and experimental results presented demonstrate the effectiveness of the proposed control concept.

1. INTRODUCTION

Vibration modes constitute an important issue in many motion control applications using conventional control techniques by limiting the system operational bounds, decreasing the control accuracy and even leading to closed loop instability. There are many examples of relevant applications such as rolling-mill drives, conveyer drives, robot drives, throttle drives for internal combustion engines and others [1]–[6]. For example, it has been reported that a conventional control structure utilising a simple PI controller cannot ensure a satisfactory performance of an electric drive with a vibration mode due to flexibility in the mechanical coupling [6]. In order to cater for such drive systems, various approaches have been developed. The application of digital filters to avoid modal excitation by the control signal is an industrial standard [4], utilising low pass filters for resonant frequencies higher than the intended control system bandwidth and band pass filters for resonant frequencies within the control system bandwidth, but with the risk of adversely affecting the dynamics of the drive. Moreover, in this approach, once
excited by external disturbances, the modal oscillations are uncontrolled due to the action of the filter in suppressing control signal components at the resonance frequencies. A better approach is active modal damping in which state variables associated with the vibrations are fed back in addition to the measured position or velocity to be controlled [5]–[9]. In all these investigations, including this one, attention is focused on the simplest representation of a flexible drive, i.e., the two mass system comprising a motor driving a balanced rigid body load via a shaft with significant torsional compliance. This system has a single vibration mode and it is possible to damp the oscillations by feeding back just one state variable in addition to the usual motor speed, such as the rate of change of the torsional deflection, but in this case the settling time of the step response cannot be set freely by choice of the feedback gains. Better results can be obtained using a control structure with the feedback of two additional state variables [7]–[9]. This is due to a complete set of state variables being used, assuming the motor inductances to have a negligible effect due to either sufficiently small time constants or the presence of current control loops. However, the measurement of some of these state variables is impractical due to the technical difficulties, excessive cost and reduction of the system reliability associated with the necessary instrumentation. Hence algorithms such as observers are necessary for estimation of these variables [8]–[10], [12]. In this paper the Kalman filter (KF) is used for this purpose.

Another issue in drive control, exacerbated by the presence of vibration modes, is significant parameter variations and uncertainties. More advanced control paradigms based on the application of non-linear [2] or adaptive [9] control concepts have been developed to ensure effective vibration damping despite these uncertainties, but due to their complexity they are still not popular in industrial applications. The alternative approach presented here circumvents this problem.

Another common requirement for motion control systems is fast counteraction of the influence of the load torque on the speed of the controlled object. Despite the presence of significant flexibility of the mechanical links between the motor and the controlled load in some cases, the approach taken for the one-mass (rigid body) system is applied in almost all of the papers presented on the control of flexible drives [11], i.e., the control structure has only one additional feedback consisting of the estimated load torque [12]. This feedback reduces the load speed transients due to changes in the load torque but is insufficient for complete elimination of the effect of the load torque changes in flexible drives. In paper [13] an additional feedback consisting of the derivative of the load speed has been proposed. Although this reduces the load speed drop it is still significant. A more advanced approach has been presented in [14] where two additional feedbacks consisting of the load torque and its first derivative are used. The authors report good dynamical characteristics of the system, but there is still room for improvement regarding minimisation of the effects of load torque changes.

The primary reason for employing FDC [16]–[18] in contrast to orthodox linear state feedback control, is that in addition to achieving vibration mode damping while
controlling the speed or position of a particular point on the load, it yields robustness properties brought about by the control law requiring disturbance inputs as well as state variables. The secondary reason for employing FDC is its applicability to plants containing continuous nonlinearities as well as linear ones, rendering it suitable for electric drives employing all types of motors. Through yielding any specified closed loop dynamics, FDC avoids tuning by trial and error, as needed for conventional drives employing PI controllers and enables the application of derivative feed-forward pre-compensators for high precision following of time-varying reference inputs in motion control applications.

2. MATHEMATICAL MODEL OF THE DRIVE SYSTEM AND PROPOSED CONTROL STRUCTURE

In this paper the commonly-used model of a drive with a flexible coupling is taken which is described by the following equations [10]:

\[
\dot{\omega}_1 = \frac{1}{T_1} (m_e - m_s)
\]

\[
\dot{\omega}_2 = \frac{1}{T_2} (m_s - m_L)
\]

\[
\dot{m}_s = \frac{1}{T_c} (\omega_1 - \omega_2)
\]

where: \(\omega_1\) – motor speed, \(\omega_2\) – load speed, \(m_e\) – motor torque, \(m_s\) – shaft (torsional) torque, \(m_L\) – disturbance torque, \(T_1\) – mechanical time constant of the motor, \(T_2\) – mechanical time constant of the load machine, \(T_c\) – stiffness time constant.

The load mass speed \(\omega_2\) is taken as the controlled variable in this investigation. The control variable \(u\) is the electromagnetic torque, \(m_e\) is the control (manipulated) variable. The control law is obtained by multiple differentiation of equation (2) for the controlled output variable \(\omega_2\), until the manipulated variable appears on the right hand side of the resulting equation. Thus substituting for \(\dot{m}_s\) using (2) to (3) yields:

\[
\dot{\omega}_2 = \frac{1}{T_2} \left\{ \frac{1}{T_c} (\omega_1 - \omega_2) \right\} - \dot{m}_L
\]

(4)

The load torque derivatives will have to remain but will be estimated by the estimator presented in [18]. Since the control variable \(m_e\) is not present in (4), the process is repeated. Thus
\[
\dot{\omega}_2 = \frac{1}{T_2} \left\{ \frac{1}{T_c} \left( \dot{\omega}_1 - \omega_2 \right) - \ddot{m}_L \right\}.
\]

(5)

Substituting for \(\dot{\omega}_1\) and \(\dot{\omega}_2\) using (1) to (2) then yields

\[
\dot{\omega}_2 = \frac{1}{T_2} \left[ \frac{1}{T_c} \left( \frac{1}{T_1} (m_e - m_s) - \frac{1}{T_2} (m_s - m_L) \right) \right] - \ddot{m}_L.
\]

(6)

Now the control variable \(m_e\) appears in (6) which is therefore the controlled output derivative equation to be used for forming the FDC law.

The order of the desired closed loop system has to be equal to the order of the controlled output derivative of (6). Here, the closed loop system will be chosen to have all three poles of magnitude, \(\omega_r\), two being complex conjugates with a damping ratio of \(\zeta_r\) and undamped natural frequency \(\omega_r\) to obtain a small overshoot in the step response if desired. With a unity d.c. gain, the closed loop transfer function is

\[
\frac{\omega_2}{\omega_z} = \frac{\omega_r^3}{s^3 + s^2 (2\xi_r \omega_r + \omega_r) + s (2\xi_r \omega_r^2 + \omega_r^2) + \omega_r^3}.
\]

(7)

The desired closed loop differential equation corresponding to (21) is then

\[
s^3 \omega_2 = \omega_r^3 (\omega_z - \omega_2) - s (2\xi_r \omega_r^2 + \omega_r^2) \omega_z - s^2 (2\xi_r \omega_r + \omega_r).
\]

(8)

The FDC law is then obtained by equating of (6) and (8) and then making \(m_e\) the subject of the resulting equation, yielding

\[
m_e = T_1 T_2 T_c \omega_r^3 (\omega_z - \omega_2) + \frac{T_1}{T_2} (m_s - m_L) + T_1 T_c \ddot{m}_L + m_s
\]

\[
+ \dot{\omega}_2 T_1 T_2 T_c \left( 2\xi_r \omega_r + \omega_r \right) + \dot{\omega}_2 T_1 T_2 T_c \left( 2\xi_r \omega_r^2 + \omega_r^2 \right).
\]

(9)

Then (22) can be rewritten as

\[
m_e = k_1 (\omega_z - \omega_2) + k_2 (m_s - m_L) + k_3 \ddot{m}_L + m_s + k_4 \dot{\omega}_2 + k_5 \dot{\omega}_2.
\]

(10)

where

\[
k_1 = T_1 T_2 T_c \omega_r^3, \quad k_2 = \frac{T_1}{T_2}, \quad k_3 = T_1 T_c,
\]

\[
k_4 = T_1 T_2 T_c \left( 2\xi_r \omega_r + \omega_r \right), \quad k_5 = T_1 T_2 T_c \left( 2\xi_r \omega_r^2 + \omega_r^2 \right)
\]

are the state feedback and load torque compensation gains. The control structure yielded by control law (10) is shown in the block diagram of Fig. 1, in which \(g_1\) and \(g_2\) are the transformation coefficients:

\[
g_1 = \frac{1}{T_2}, \quad g_2 = \frac{1}{T_2 T_c}.
\]

(12)
As can be seen in Fig. 1, the control law requires non-measured state variables, i.e., the shaft torque and the load speed, as well as non-measurable disturbance signals comprising the load torque together with its first and second derivatives. It is evident that the feedbacks from the load torque and its derivatives with gains, $k_2$ to $k_4$, and $g_1$, $g_2$ defined by (26) introduced by FDC exactly compensates the load torque acting on the load mass.

The main-path transfer function of the drive system with elastic joint can be formulated as:

$$G_p(s) = \frac{\omega_2}{\omega_z} = \frac{k_1}{s^3T_1T_c - s^3T_2T_c g_2 k_4 + s(T_1 + T_2 - T_2(1 + k_2 + g_1 k_3)) + k_1}.$$  \hspace{1cm} (13)

The disturbance-path transfer function of the drive system with elastic joint, without additional feedbacks from load torque and its derivatives) takes form:

$$G_z(s) = \frac{\omega_2}{m_L} = \frac{-s^2T_1T_c + sT_c g_2 k_4 + k_2 + g_1 k_5}{s^3T_1T_2T_c - s^2T_2T_c g_2 k_4 + s(T_1 + T_2 - T_2(1 + k_2 + g_1 k_5)) + k_1}.$$  \hspace{1cm} (14)

The compensation of disturbance-path transfer function zeros is obtained by zeroing the nominator of the transfer function (14). As can be seen form the schematic diagram of the drive system (Fig. 1), feedbacks from the load torque and its first and second derivatives are introduced, with values equaling to zeros the following expressions in the nominator of the mentioned transfer function) So the resulting disturbance-path transfer function of the system is:

$$G_z(s) = \frac{\omega_2}{m_L} = \frac{-s^2(T_1T_c - k_3) + s(T_c g_2 k_4 - g_1 k_4) + (k_2 + g_1 k_5 - k_2 - g_1 k_5)}{s^3T_1T_2T_c - s^2T_2T_c g_2 k_4 + s(T_1 + T_2 - T_2(1 + k_2 + g_1 k_5)) + k_1}.$$  \hspace{1cm} (15)
By a simple substitution of the gains in the nominator of (15) with values given by (11), we obtain zeroing the nominator of (15), what means, that the two-mass drive speed is insensitive to the load torque change.

3. SIMULATION STUDY

In the FDC structure with direct feedbacks from all state variables, the load speed transient has the desired shape only when the electromagnetic torque limit is neglected (Fig. 2a, b). The rising time and overshoot depend on the assumed value of the resonant frequency $\omega_r$ and the damping coefficient $\xi_r$. When the load torque is applied

Fig. 2. Simulated transients of the motor and load speeds (a, c) and electromagnetic, shaft and load torques (b, d) for assumed resonant frequency of the system $\omega_r = 45 \text{ s}^{-1}$ for the system without (a, b) and with (c, d) limitation of the electromagnetic torque

Rys. 2. Przebiegi prędkości silnika i obciążenia (a, c) oraz momentów elektromagnetycznego, skręconego i obciążenia przy założonej wartości pulsacji rezonansowej układu $\omega_r = 45 \text{ s}^{-1}$ w układzie bez (a, b) oraz z uwzględnieniem ograniczenia momentu elektromagnetycznego (c, d)
(at \( t = 0.4 \) s), the control structure forces a very big value of the electromagnetic torque (Fig. 2b) and removes this disturbance from the load speed completely. This confirms the efficiency of the control law based on the FDC theory. However, such a huge value of torque is not possible in the real system. When the electromagnetic torque limit is set to 3 [p.u.], the shape of the load speed is visibly affected by the load torque disturbance (Fig. 2c, d).

Since a FDC structure (Fig. 1) has no integration term, the shape of the load speed, observed in Fig. 4a, c, can be improved using an additional memory element presented in Fig. 3. Operating principle of this memory element relies on memorizing the part of the forced value of the electromagnetic torque higher that the assumed limit \( m_{e,max} \). Then, the resultant value is added to the value of \( m_{ez}^* \) of the electromagnetic torque when this signal \( (m_{ez}^*) \) is smaller than the assumed limit. In this case the whole torque given to the drive equals \( m_{ez} = m_{ez}^* + m_{ed} \). In order to ensure good dynamics of the system, the selection of a suitably high value \( k_w \) is necessary. In Figure 4 transients of the analyzed drive system with limitation of the electromagnetic torque, with an additional memory element, described above, are shown.

![Fig. 3. The block diagram of the additional memory element](image)

**Rys. 3 Schemat blokowy dodatkowego elementu pamiętającego**

![Fig. 4. Simulated transients of the motor and load speeds (a) and electromagnetic, shaft and load torques (b) for the system with additional memory element](image)

**Rys. 4. Przebiegi prędkości silnika i obciążenia (a) oraz momentów elektromagnetycznego, skrętnego i obciążenia w układzie z dodatkowym elementem pamiętającym**
The application of the additional memory element improves the load speed transient significantly. The value of the speed fall has been reduced about six times.

\[ \omega_2 = 0.5 \text{ [p.u.]} \] (Fig. 5a). The proper dynamics of the drive can be ensured only by the control structure with all...
feedbacks: from the load torque and its derivatives simultaneously – Fig. 4c. In such case the torsional torque tracks the forced electromagnetic torque perfectly.

4. EXPERIMENTAL STUDY

The FDC structure with Kalman filter [18] has been examined in the laboratory set-up, composed of two converter-fed DC machines (500 W each) coupled by the elastic shaft, described in detail in [18]. The nominal parameters of the drive are $T_1 = T_2 = 203$ ms, $T_c = 2.6$ ms. The control structure working with the Kalman filter is implemented in a digital signal processor using the dSPACE 1102 card.

Fig. 6. Experimental transients of the motor and load speeds (a, c) and electromagnetic and shaft torques (b, d) for the system without (a, b) and with additional feedbacks (c, d) from the derivatives of the load torque.

Rys. 6. Przebiegi prędkości silnika i obciążenia (a, c) oraz momentów elektromagnetycznego i skrętnego w układzie bez (a, b) oraz z (c, d) dodatkowymi sprzężeniami od pochodnych momentu obciążenia.
First the control structure is investigated under reverse operation, with rapidly changing load torque. The additional feedbacks from the load torque derivatives, provided by the developed estimator, improve the drive performance and eliminate the effect of the load torque changes from the load machine speed. In Fig. 6 transients of the system without (a, b) and with the additional feedback from the load torque derivatives are presented. The drive starts working without additional load torque. At the time $t_1 = 0.4$ s the load torque is switched on. Then, after reversal, at the time $t_2 = 1.4$ s it is switched off again. The described cycle of the operation is repeated after $t_3 = 2$ s. When the structure works without additional feedbacks from the load torque derivatives (Fig. 6a, b), the rapid changes of the load torque influence the load speed transient in visible way, the system needs 300 ms to eliminate the steady-stay error. The application of additional feedbacks from the load torque derivatives improves the system performance significantly (Fig. 6c, d). The steady-stay error is

![Graphs showing system transients](image)

Fig. 7. Transients of the motor and load speeds (a, c) and electromagnetic and shaft torques (b, d) for the system without (a, b) and with additional feedbacks (c, d) from the load torque and its derivatives.

Rys. 7. Przebiegi prędkości silnika i obciążenia (a, c) oraz momentów elektromagnetycznego i skrętnego w układzie bez (a, b) oraz z (c, d) dodatkowymi sprzężeniami od momentu obciążenia i jego pochodnych.
eliminated after 100 ms. During reversal the shape of the load speed is affected by the passive torque, the reference value is reached after approximately 400 ms. In the structure with the additional feedbacks from the load torque and its derivatives the shape of the load speed results from the design parameters (resonant frequency and damping coefficient).

The advantage of the proposed control structure is more clearly visible in the systems where the load torque is not changing so rapidly. In those cases the estimator can reconstruct the values of the first and the second derivatives of the load torque properly. To illustrate this, the control system with sinusoidal-shape changeable load torque is tested. The system transients for the reference speed set to 50% of the nominal value are presented in Fig. 5. The speed of the system without additional feedbacks from the load torque derivatives is varying (Fig. 5a, b) according to the load torque variation. The insertion of the feedback from the load torque first derivative reduces speed oscillations, but significant improvement of the drive performance is achieved when all additional feedbacks from the load torque derivatives structure are used (Fig. 5c, d).

5. CONCLUSIONS

In the paper an application of the FDC-based control structure for two-mass drive system has been presented. This control structure ensures independent location of the system closed-loop poles and cancellation of the system zeros. This allows removing the influence of the load torque on the load machine speed. The characteristic of the proposed control structure working with the rapid-changing load torque can be improved by application additional memory elements. It reduces the speed fall due to the changes of the load torque significantly. The performance of the control structure depends strongly on the applied state estimator which provides the information of the non-measurable state variables. Due to the finite speed of the state estimator, the proposed control structure is especially designated for the industrial systems with a slowly-varying load torque. The experimental results more confirmed theoretical considerations and showed the effectiveness of the proposed control concept.

REFERENCES


