MODELING OF GENERATOR PERFORMANCE OF BLDC MACHINE USING MATHEMATICA SOFTWARE

In this article three different circuit models of BLDC motor are compared: Lagrange energetic model, arbitrary reference frame model and multiple reference frame model. Simulations of generator performance were analyzed. All three models were developed in Mathematica 5.2 software.

1. INTRODUCTION

Year after year technological capabilities of building BLDC machines of grater power and rotation speed as long as their applications are increasing. This constructions owns their popularity to high electromagnetic torque to rotor inertia ratio and power to weight ratio. BLDC machines are also very to control.

Very important is development of new models of this machines. One which allows fast result receiving and on the other hand will be neglecting as small as it is possible of reality. Models which allow real time diagnostic of machine.

2. MODELLING OF BRUSHLESS DC MACHINE

1.1. PHYSICAL MODEL

This paper presents modelling of surface mounted permanent magnets BLDC machine (fig. 1) presented in [2] and [5]. Back EMF generated in stator windings by PM excitation field is trapezoidal. Stator is fitted with 3-phase symmetrical winding, in which beginnings of stator coils are described as \( a_s_1 \) \( a_s_2 \), endings of stator coils are
described as \((a_{s_1}', a_{s_2}')\). Because of PM number of model variables can be smaller than in standard wound field machine. All presented models are neglecting the effects of magnetic saturation, saliency, hysteresis, cogging torque.

Fig. 1. BLDC model in stator \((as, bs, cs)\) and rotor \((d, q)\) axes, where: \(F_s, F_r\) – stator and rotor magneto motive force (MMF), \(\omega_r\) – electrical rotation speed, \(\omega_{rm}\) – mechanical rotation speed, \(T_e\) – electromagnetic torque, \(T_l\) – load torque, \(J\) – rotor inertia, \(B_m\) – mechanical dumping due to friction, \(\theta_{rm}\) – rotor position

2.2. LAGRANGE ENERGETIC MODEL (MODEL-1)

Lagrange energetic model is developed as shown in [4], and is based on very simple circuit in which all stator coils are reduced to be represented by magnetic elements where: \(L_s\) is self and \(M_s\) is mutual inductance and by dissipative elements, where: \(R_s\) is stator winding resistance. The load is represented by magnetic elements where: \(L_l\) is self inductance and by dissipative elements, where: \(R_l\) is load resistance. \(R_n\) is representing connection of generator neutral point to load supply neutral point.

Lagrange function in generalized variables \((i_{as}, i_{bs}, i_{cs}, \theta_{rm}, \omega_{rm})\):

\[
L_{tot} = \frac{1}{2} i_{as}^2 L_s + i_{as} \lambda_{m}^s \left( \theta_{rm} - \frac{2}{3} \pi \right) + i_{bs} (i_{bs} - i_{as}) M_s + \frac{1}{2} (i_{bs} - i_{as})^2 L_s + \frac{1}{2} (i_{bs} - i_{as}) M_s + \frac{1}{2} (i_{bs} - i_{as})^2 L_s + \frac{1}{2} (i_{bs} - i_{as}) M_s
\]

\[
+ \frac{1}{2} i_{cs}^2 L_r(t) + \frac{1}{2} (i_{cs} - i_{as}) L_r(t) + \frac{1}{2} (i_{cs} - i_{as})^2 L_r(t) + \frac{1}{2} (i_{cs} - i_{as}) L_r(t) + \frac{1}{2} i_{as} \lambda_{rm}^s \left( \theta_{rm} + \frac{2}{3} \pi \right) + \frac{1}{2} i_{bs} \lambda_{rm}^s \left( \theta_{rm} + \frac{2}{3} \pi \right) + \frac{1}{2} i_{cs} \lambda_{rm}^s \left( \theta_{rm} + \frac{2}{3} \pi \right) + \frac{1}{2} \omega_{rm} \lambda_{rm}^s
\]
and the Raleigh dissipation function in generalized variables \((i_a, i_b, i_c, \omega_{rm})\):

\[
P_{R\lambda}(\omega_{rm}) = \frac{1}{2} R_i i_a^2 + \frac{1}{2} R_i (i_b - i_a)^2 + \frac{1}{2} R_i (i_c - i_a)^2 + \frac{1}{2} R_i (i_c - i_b)^2 + \frac{1}{2} R_i (i_c - i_a)^2 + \frac{1}{2} R_i (i_c - i_b)^2 + \frac{1}{2} R_i (i_c - i_a)^2 + \frac{1}{2} R_i (i_c - i_b)^2 + \frac{1}{2} D \omega_{rm}^2
\]  

(2)

The complete Lagrange energetic model can be derived from:

\[
\begin{align*}
\frac{d}{dt} \frac{\partial \mathcal{L}_{\text{net}}}{\partial i_a} - \frac{\partial \mathcal{L}_{\text{net}}}{\partial q_a} + \frac{\partial P_{\text{Rad}}}{\partial i_a} &= v_{as} - v_{bs} \\
\frac{d}{dt} \frac{\partial \mathcal{L}_{\text{net}}}{\partial i_b} - \frac{\partial \mathcal{L}_{\text{net}}}{\partial q_b} + \frac{\partial P_{\text{Rad}}}{\partial i_b} &= v_{bs} - v_{cs} \\
\frac{d}{dt} \frac{\partial \mathcal{L}_{\text{net}}}{\partial i_c} - \frac{\partial \mathcal{L}_{\text{net}}}{\partial q_c} + \frac{\partial P_{\text{Rad}}}{\partial i_c} &= v_{cs} \\
\frac{d}{dt} \frac{\partial \mathcal{L}_{\text{net}}}{\partial \omega_{rm}} - \frac{\partial \mathcal{L}_{\text{net}}}{\partial \theta_{rm}} + \frac{\partial P_{\text{Rad}}}{\partial \omega_{rm}} &= T_i
\end{align*}
\]  

(3)

2.3. ARBITRARY REFERENCE FRAME MODEL (MODEL-2)

The arbitrary reference frame model based on R. H. Park transformation presented in [3], with assumption that back EMF (BEMF) is sinusoidal. Voltage equations in rotor reference-frame (arbitrary reference frame) variables:

\[
v_{qd0s}^r = r_{i}^r v_{q0s}^r + \omega \lambda_{dq0}^r + \frac{d \lambda_{qd0s}^r}{dt}
\]  

(4)

where:

\[
\lambda_{qd0s}^r = \begin{bmatrix}
\lambda_{dq}^r(t) \\
\lambda_{dq}^r(t) \\
\lambda_{dq}^r(t)
\end{bmatrix} = L_{i}^r i_{q0s}^r + \lambda_{m}^r 1 0
\]  

(5)

and the mechanical equation:

\[
T_e - T_i = \left( \frac{P}{2} \right) J \omega_s + \left( \frac{P}{2} \right) B_m \omega_s
\]

\[
T_e = \left( \frac{3}{2} \right) \left( \frac{P}{2} \right) \left( \lambda_{dq}^r i_{dq}^r - \lambda_{dq}^r i_{ds}^r \right)
\]  

(6)
2.4. MULTIPLE REFERENCE FRAME MODEL (MODEL-3)

As it is elaborated in article [1] this model is similar to one in previous section except assumption that BEMF is sinusoidal. It is necessary to represent PM excitation flux coupled with stator coils as Fourier series [6] because of non sinusoidal character of stator BEMF. Assuming that BEMF is half wave symmetric there are only odd elements. This allows us to crate stator flux linkage equations:

\[ \lambda_{m}^{r} (\theta_r) = \lambda_{m}^{r} \sum_{n=1}^{\infty} \left( K_{(2n-1)} \sin\left( (2n-1)\theta_r \right) \right) \]  \hspace{1cm} (7)

Voltage equations in multiple reference frame model are exactly the same as in arbitrary reference frame. Only difference is in description of mutual stator and rotor flux:

\[ \lambda_{qz} = L_q i_q + \lambda_m \sum_{n=1}^{\infty} \left( K_{(6n-1)} + K_{(6n+1)} \right) \sin(6n \theta_r) \]

\[ \lambda_{ds} = L_d i_d + \lambda_m \sum_{n=1}^{\infty} \left( K_{(6n-1)} - K_{(6n+1)} \right) \sin(6n \theta_r) \]  \hspace{1cm} (8)

\[ \lambda_{qs} = L_q i_{qs} + \lambda_m \sum_{n=1}^{\infty} K_{(6n-3)} \sin((6n-3) \theta_r) \]

and the electromagnetic torque is defined as:

\[ T_e = \frac{3}{2} \frac{P}{2} \lambda_{m}^{r} \left\{ i_{qz} \left( 1 + \sum_{n=1}^{\infty} \left( K_{(6n-1)} + K_{(6n+1)} \right) \cos(6n \theta_r) \right) + \\
+ \sum_{n=1}^{\infty} \left( K_{(6n-1)} + K_{(6n+1)} \right) \sin(6n \theta_r) + \\
+ 2i_{ds} \sum_{n=1}^{\infty} K_{(6n-3)} \sin((6n-3) \theta_r) \right\} \]  \hspace{1cm} (9)

3. SIMULATION

3.1. STEP CHANGE OF GENERATOR LOAD CHARACTERISTICS

Figure 2 shows dynamic characteristic of BLDC generator during step change of load in \( t = 2.5 \) [s]. Difference can be seen in generated current between models 1 and 3 (which are similar) and model 2. Current generated in this simulation of MODEL-2 is sinusoidal and in plots from other two models is similar to a trapezoid. That results from the fact that MODEL-1 and MODEL-3 have defined excitation flux as Fourier series and the derivative of this series is trapezoid.
3.2. SYMMETRICAL 3-PHASE FAULT CHARACTERISTICS

Figure 3 shows dynamic characteristic of generator during symmetrical line-to-line fault. This fault is represented by rapid change in time $t = 2.5 \, [s]$ of load resistance to zero. In characteristic of phase current it can be seen that the most unusual is solution of MODEL-3 where the influence of higher harmonics is more significant than in MODEL-1. That can suggest that MODEL-1 and MODEL-2 are better in this case. Nothing further from the truth, when it’s clear that mechanical characteristics solutions of MODEL-1 and MODEL-3 are practically alike. Moreover aperiodic component of short circuit current can be observed.

4. CONCLUSIONS

Presented models were developed in Mathematica 5.2 software [7]. This software is similar to other mathematical application such as MathCAD or Matlab. It has very extended graphical engine witch is helpful in presenting results of calculations.

It is impossible to indicate which of those three models is the best. It depends on outcomes we expect from circuit model of electric machine. When we want detailed characteristics and not complicated model to simulate not natural behaviour of motor we would chose Lagrange energetic model (MODEL-1). However if we want to receive solution quickly Arbitrary reference frame model (MODEL-2) is a good choice. Multiple reference frame model (MODEL-3) is a solution between MODEL-1 and
MODEL-2, where one gets result of simulation faster then in MODEL-1 and more precise than in MODEL-2.

REFERENCES


SYMULACJA STANU PRACY PRĄDNICOWEJ MASZyny BEZSZCZOTKOWEj PRąDU STAŁEGO Z MAGNESAMI TRWAŁYMI W PROGRAMIE MATHEMATICA