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DETERMINING EQUIVALENT PARAMETERS OF A MAGNETIZING RLC SYSTEM FOR APERIODIC CURRENT WAVESHAPES

The paper presents calculation procedures serving to determine equivalent parameters of systems for magnetizing permanent magnets – inclusive magnets installed in electric machines. The term “equivalent parameters of RLC systems” denotes values of resistance of resistive elements, of winding inductance and of capacitance of capacitors, with additional consideration imperfections of those elements, i.e. resistance of leakage, inductance of capacitors as well as the inductance of resistive elements. If the effect of the imperfection of the above mentioned system components is not taken into account then the accuracy of calculations of the \(i(t)\) magnetising currents is deteriorated. It is possible to calculate the values of currents \(i(t)\) being close to the measured values, but when the parameters of the equivalent RLC circuits are used. The method of determining the equivalent parameters is presented below.

1. INTRODUCTION

The impulse-type RLC magnetizing system is composed of in series subassemblies connected in series: a capacitor bank \(C\), magnetizing winding of resistance \(R_u\) and inductance \(L_u\) and of the components connecting the individual subassemblies (inclusive a connector in the form of an ignitron, thyristor or transistor) with resistance \(R_a\). Each of these subassemblies can be characterized by the value of the basic parameter mentioned above and a parameter resulting from the imperfection of the subassembly. The parameters resulting from the imperfection of the subassemblies become appreciable during the dynamic operation of the system. Therefore it is necessary to use equivalent parameters in calculations of the function \(i(t)\). Thus, it is necessary to

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transform suitable the equation describing the course of discharge of electric energy in the \( RLC \) system. In the sequel, the resistance components \( R_u \) and \( R_a \) are treated together.

It is known from the general theory of electric circuits [1] that the solution of the differential equation, mentioned above, describing an \( RLC \) circuit connected in series, supplied from a DC source, has, in the case of an aperiodic discharge, the exponential form. The current waveform is as follows:

\[
i(t) = \frac{U_k}{\omega_a L} e^{-\beta t} \sinh(\omega_a t) \quad (1)
\]

where: \( \beta = R/2L \), \( \omega_a = \sqrt{\beta^2 - \omega_0^2} \) are attenuation coefficient and angular frequency of the damped course, respectively; \( \omega_a < \beta \) depends on the resonance angular frequency \( \omega_0 = 1/\sqrt{LC} \) and the attenuation coefficient. \( U_k \) is terminal voltage, here the voltage of a capacitor bank.

2. INITIAL MATHEMATICAL MODEL

The following relationship presents a more useful form of the expression (1):

\[
i(t) = \frac{U_k}{2\omega_a L} e^{-(\beta-\omega_a)t} - \frac{U_k}{2\omega_a L} e^{-(\beta+\omega_a)t}
\]

Superimposing the value of the derivative \( di(t)/dt = 0 \) we obtain the value of the time which passes from the moment when the discharge of the capacitors is initiated to the moment when the power reaches the peak value.

\[
\tau = \frac{1}{2\omega_a} \ln \frac{\beta + \omega_a}{\beta - \omega_a} \quad (2)
\]

where \( \tau \) is time constant.

Marking on the graph \( i(t) \) (determined by measurements) the values of current \( i_1, i_2, i_3, ..., i_n \) (see Fig. 1) which correspond to the multiplication of time \( \tau, 2\tau, 3\tau, ..., m\tau \), a simple relationship, which occurs between the parameters of the \( RLC \) system and the current, can be noted. This relationship can be expressed by formula:

\[
i(m\tau) = \frac{U_k}{2\omega_a L} \left( \frac{\beta + \omega_a}{\beta - \omega_a} \right)^m \frac{\beta - \omega_a}{2\omega_a} - \frac{U_k}{2\omega_a L} \left( \frac{\beta + \omega_a}{\beta - \omega_a} \right)^m \frac{\beta + \omega_a}{2\omega_a} \quad (3)
\]
Owing to the above and the registered waveform of the current and of the initial value of voltage on the capacitance, it is possible to determine the $R$, $L$, $C$ parameters of the circuit in which this current appeared. It is sufficient to proceed as follows:

1. to read the value of voltage on capacitance at the moment just before discharge,
2. to determine the time $\tau$ after which the current reaches its peak value,
3. to determine the value that denoted in the figure by $i_1 = i_{\text{max}}$,
4. to repeat determination of the value of current for the times $2\tau$ and $3\tau$,
5. to proceed calculations according to the following relationships.

After adopting the coefficient $\kappa$ equal to the quotient of the aperiodic waveform pulsation by its attenuation

$$\kappa = \frac{\omega_0 \beta}{\beta}; \kappa \in (0,1)$$  \hspace{1cm} (4)

we obtain a more clear notation of the dependence of current on time. Using this substitution, the time to reaching the maximum value of the current is given by formula:

$$\tau = \frac{1}{\beta} \frac{1}{2\kappa} \ln \frac{1 + \kappa}{1 - \kappa}$$  \hspace{1cm} (5)

This time is a function of two parameters: the attenuation $\beta$ and the coefficient $\kappa$. The part of this formula depending only on the coefficient $\kappa$ can be represented by the function of this single parameter, denoted here as function $\xi$. 
The value of the coefficient $\kappa$ is adopted initially on the basis of the value determined using expression $\kappa = \omega_a / \beta$.

The value of the resonance pulsation of the aperiodic waveshape $\omega_a$ and the value of the attenuation coefficient are calculated on the basis of the rated values of the parameters of the magnetizing system subassemblies $RLC$. An example of the variation of $\zeta(\kappa)$ is shown in Fig. 2.

![Graph showing the relationship $\zeta(\kappa)$](image)

Fig. 2. Relationship $\zeta(\kappa)$

Introducing the following substitution in order to make the formulae simpler

$$a = \frac{U_i}{2\omega_a L} = \frac{U_i}{2\beta \kappa L} \quad (7); \quad g(\kappa) = \left(\frac{1+\kappa}{1-\kappa}\right)^{\frac{1-\kappa}{2\kappa}} \quad (8) \quad h(\kappa) = \left(\frac{1+\kappa}{1-\kappa}\right)^{\frac{1+\kappa}{2\kappa}} \quad (9)$$

one can, using formula (3), obtain a set of expressions for the first three values of $m$:

$$i_1 = ag - ah; \quad i_2 = ag^2 - ah^2; \quad i_3 = ag^3 - ah^3 \quad (10)$$

Using these expressions, it is possible to determine the values of functions $a, g$ and $h$.

$$a = \frac{i_1^2}{\sqrt{4i_1 i_3 - 3i_2^2}} \quad (11); \quad g = \frac{ai_2 + i_1^2}{2ai_1} \quad (12); \quad h = \frac{ai_3 - i_1^2}{2ai_1} \quad (13)$$
Then the value of the coefficient $\kappa'$ shall be determined:

$$\kappa' = \log_{(gh)} \left( \frac{h}{g} \right)$$  \hspace{1cm} (14)

The coefficient $\kappa'$ shall be calculated as the logarithm (with the basis equal to the product of the function $g$ and $h$) from the quotient of the functions $h$ and $g$.

After finding the value of the coefficient $\kappa'$, we have got the determined value of the function $\xi$:

$$\xi = \frac{1}{2\kappa'} \ln \frac{1+\kappa'}{1-\kappa'}$$  \hspace{1cm} (15)

This value permits to determine the coefficient of attenuation and the angular frequency

$$\beta = \frac{\xi}{\tau}$$  \hspace{1cm} (16); \hspace{1cm} $$\omega_0 = \kappa' \beta$$  \hspace{1cm} (17)

Now it remains, only, to determine the basic ideal parameters of the circuit in which the current was generated:

$$L = \frac{U_k}{2a\beta \kappa'}$$  \hspace{1cm} (18) \hspace{1cm} $$C = \frac{1}{(\beta^2 - \omega_0^2)L}$$  \hspace{1cm} (19) \hspace{1cm} $$R = 2\beta L$$  \hspace{1cm} (20)

In such a way, new $R', L', C'$ parameters of the magnetizing system were determined. The wave front of the magnetizing current is reflected on the values of these parameters. The current course calculated on the basis of such determined parameters corresponds better to the experimental curve. It should be mentioned that in the case of the aperiodic critical discharge, i.e. when $(R/L)^2 = 4/LC$, [2], [3], [5], somewhat different relationships will hold. The case of determining equivalent parameters for a periodic discharge is presented in [4]. Computer calculations proceeded for both aperiodic and periodic cases indicate the significant improvement in accuracy of determining the current curve when using the values of equivalent parameters.

3. SUMMARY

When analyzing the calculation procedures serving for determining the equivalent parameters for cases of aperiodic discharges in $RLC$ systems, supported by calculations and measurements, it can be stated that:
there is a possibility to determine equivalent parameters which can be applied in calculations of the $i(t)$ curve,
application of equivalent parameters in calculations of the current curve, that is determined during magnetizing permanent magnets, permits to improve the calculation accuracy of this curve.

REFERENCES


WYZNACZANIE ZASTĘPCZYCH PARAMETRÓW UKŁADU MAGNESOWANIA RLC DLA APERIODYCZNEGO PRZEBIEGU PRĄDU

W opracowaniu przedstawiono procedury obliczeniowe służące do wyznaczania zastępczych parametrów układów do magnesowania magnesów trwałych. Pod terminem 'parametry zastępcze układu RLC' rozumie się wartości rezystancji elementów oporowych, indukcyjności uzwojeń i pojemności kondensatorów z dodatkowym uwzględnieniem niedoskonałości tych elementów, a więc rezystancji upływu i indukcyjności kondensatorów, a także indukcyjności elementów oporowych. Nie uwzględnienie wpływu wymienionych niedoskonałości elementów układu powoduje zmniejszenie dokładności obliczeń przebiegów magnesujących $i(t)$. Możliwe jest obliczanie przebiegów $i(t)$ bliskich wartościom pomierzonym, jednakże przy zastosowaniu parametrów zastępczych układów RLC. Wyzwier opisano sposób wyznaczania parametrów zastępczych.